Clustering Irregular Spaced LiDAR TINs for 3D Reconstruction

Nicholas S. Shorter, Takis Kasparis, Michael Georgiopoulos, and Georgios C. Anagnostopoulos

Abstract—**Several sets of features, existent in triangulated, irregularly spaced LiDAR data, are extracted, conditioned, and presented to a number of clustering algorithms with the intent to recognize planar structures within the data. From those planar structures, encoded by the clustering algorithms, 3D models are then reconstructed. The purpose of this paper is to evaluate the performance of these clustering algorithms' ability to accurately cluster coplanar triangles into groups correlating to a given, depicted structure's roof planes. Several preprocessing, input conditioning procedures are presented. Also, a post processing planar regression algorithm is implemented to further refine the clustering algorithms' results to realize 3D reconstructed models of the LiDAR points. Furthermore, membership criterions, for a given triangle to correctly belong to a roof cluster, are proposed. Measures in which to evaluate the performance of the clustering algorithms ability to accurately encode the triangulated LiDAR data are also proposed.**

I. Introduction

Light Detection and Ranging (LiDAR) systems sample a given terrain into a finite collection of irregularly spaced data points. A triangulation algorithm can then adaptively connect the data points to represent the depicted terrain with a series of interconnected, non overlapping triangles or a triangulated irregular network (TIN). The TIN can then be analyzed and coplanar triangles can be clustered into groups corresponding to the depicted buildings' roof planes. Then extracting these roof planes and merging them can lead to reconstructing the sampled, original, irregularly spaced, data points into three dimensional complete models. This is all to be done autonomously, that is, with no user intervention.

Several military and commercial applications exist for 3D reconstruction from LiDAR data. For the military, the analysis of LiDAR data can be used for target recognition applications. Work in training volume correlation filters (VCFs) to recognize tanks and other military vehicles within LiDAR data has recently been developed [6]. There is investigation underway for mounting LiDAR sensors on unmanned aerial vehicles (UAVs) [5]. This would enable aerial surveying of terrains in which military forces were denied access too. Scenes surveyed by an UAV or high flying plane with a LiDAR sensor could then be reconstructed into 3D models. The analysis of 3D models of given terrains has a variety of commercial applications: urban planning; network planning for mobile communication, spatial analysis of air pollution and noise nuisances, geographical information systems, and security services.

Regarding buildings with relatively simple structures, it may be possible, with enough conditioning and preprocessing of the input data and post processing of the clustered results, to make almost any clustering algorithm capable of appropriately clustering the triangles into groups correlating to a given building's roof planes. It is the purpose of this paper to investigate which clustering algorithms are most efficient for accurately clustering coplanar triangles into their correct, corresponding groups for both simple and complex buildings without having to change any of the algorithm parameters regardless of the extremity of the building complexity.

II. Problem Formulation

A. LiDAR TIN Clustering

There is a plethora of clustering algorithms documented in the literature [13]. To optimize the performance of a chosen clustering algorithm, it is imperative to select and precondition features from the data set which cause data points of the same cluster to appear close to one another in the input space from the perspective of a given clustering algorithm. Furthermore, a clustering algorithm should be chosen in which possesses the ability to accurately recognize the clusters existent in the data with an appropriate pattern encoding mechanism.

Representing the LiDAR data set with a TIN has several advantages. Because none of the triangles existent in the TIN overlap and none of the edges intersect, all of the triangles are uniquely defined. Furthermore, during construction of the TIN, the indices of the triangles adjacent to a given triangle are stored in that triangle's data structure. This enables one to implement spatial and connectivity analyses. The irregular LiDAR points are *not* interpolated due to the fact that doing so introduces interpolation errors. Garland and Heckbert's Greedy Insertion Triangulation Algorithm [3] is therefore chosen to triangulate the raw LiDAR data because of the algorithm's accuracy and ability to triangulate irregularly spaced data.

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N. S. Shorter is a PhD Electrical Engineering Student at the University of Central Florida, FL, 32816 USA (corresponding author to provide phone: 407-882-2096; e-mail: nshorter@mail.ucf.edu; http://www.nshorter.com).

Dr. Takis Kasparis is with the department of Electrical and Computer Engineering, University of Central Florida, Orlando FL, 32816 USA (e-mail: kasparis@mail.ucf.edu).

Dr. Michael Georgiopoulos is with the Electrical and Computer Engineering Department, University of Central Florida, Orlando, FL 32816 USA (e-mail: michaelg@mail.ucf.edu).

Dr. Georgios C. Anagnostopoulos is with the department of Electrical and Computer Engineering, Florida Institute of Technology, Melbourne, FL 32901 USA (email: georgio@fit.edu).

B. Triangle Membership to a Roof Plane Cluster

Before selecting a clustering algorithm, it is necessary to formulate what exactly constitutes a coplanar triangle cluster correlating to a building roof plane. Ideally, triangles belonging to the same roof plane in a given building structure share the following attributes:

1. *Coplanarity* – Given a planar equation of the following form:

$$a \cdot x + b \cdot y + c \cdot z + d = 0 \qquad \textbf{(1)}$$

   All coplanar triangles belonging to a given roof cluster should fall close to that plane within a certain threshold, defined as follows (the perpendicular distance of a given point from that established plane):

$$D = \frac{|a \cdot (x_0 - x_1) + b \cdot (y_0 - y_1) + c \cdot (z_0 - z_1)|}{\sqrt{a^2 + b^2 + c^2}} \qquad \textbf{(2)}$$

2. *Uniform normal vector*- The normal vectors of all common roof plane triangles should have roughly the same orientation
3. *Vertex Elevation Difference* - For common roof plane triangles, the maximum difference in elevation between any two vertices of those triangles should be less than a threshold value (see MX_{zdiff} in Fig. 1).
4. *Pitch* - (Related to 3) The pitch, in the spherical coordinate representation of the normal vector, should be less than 60 degrees ($\theta \leq 60$) from the z-axis, where the pitch is defined depicted as follows:

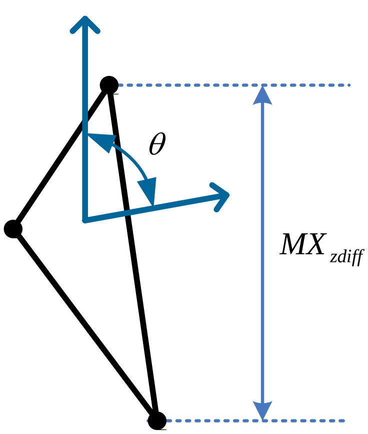

Fig. 1 - Pitch of Roof Plane (Theta)

5. *Connectivity* – A triangle is defined as adjacent to another triangle in a TIN if those triangles share a common edge. All roof plane triangles, in a given roof cluster, should be interconnected with one another via a series of adjacent, connected triangles.

The coplanarity, uniform normal vector (related to the pitch) and connectivity have been previously outlined in the literature [5], [11]. However, the authors are unaware anyone listing the vertex elevation difference.

C. Filtering out noise in the LiDAR TIN

Due to the limited accuracy of the LiDAR sensor itself, the noise incurred from the emitted laser propagating through the earth's atmosphere, and other noise influences ([3], [11], [15]), the actual LiDAR data (Fig. 3) deviates from what would be considered ideal data points (Fig. 2).

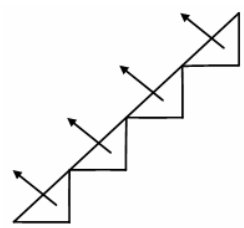
Fig. 2 Ideal TIN Vectors

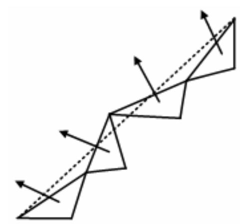
Fig. 3 Actual TIN Vectors

In order to deal with the inherent noise in the data set, a modification to the Greedy Insertion Triangulation algorithm which realizes a noise filtering technique is implemented during triangulation. This noise filtering technique merges points belonging to roof planes which fall below a certain adjustable threshold (see [9],[10] for further elaboration on this technique).

D. Clustering Algorithms Used to Encode the LiDAR TIN

Several clustering algorithms, because of their respective key characteristics, were selected to do the TIN clustering performance comparison. Baraldi and Parmiggiani's Fuzzy Simplified Adaptive Resonance Theory (FSART) algorithm [2] was chosen because its long term memory templates correspond to the ideal orientation vector of a given encoded roof plane when the inputs are presented as the triangles' normal vectors in spherical coordinates (for more information see [9],[10]). Also, the commutative properties of FSART's activation function help lessen any effects in which the order that the data is presented to the algorithm can have on the algorithm's performance [2].

Williamson's Gaussian ART (GA) was another clustering algorithm chosen for the comparison, specifically because GA's performance is dependent on the amount of noise present rather than how the noise is distributed [13].

To compare with the performance of GA and FSART, Carpenter and Grossberg's Fuzzy ART (FA) algorithm [1] was also implemented. While GA uses Gaussian distributions to encode its categories, FA uses hyper-rectangles to encode its categories.

III. Experimental Procedures

A. Data Set Specifications

The Fairfield data set, used for this research effort, covers two square kilometers of both an urban and residential scene in Fairfield, Australia. The LIDAR sensor used to procure this data set was able to capture, for each laser point, the longitude, latitude, elevation, first return pulse, last return pulse and returned laser intensity. The procured LIDAR data has an approximate point density of 1 point per 1.3m^2 and the aerial photograph, accompanying the LiDAR data captured, has 15 centimeter pixel resolution.

B. Optimizing Clustering Algorithm Adjustable Parameters

All of the selected algorithms have a user defined parameter known as the vigilance parameter: a parameter which controls how conservative that algorithm is when accepting new patterns to a pre-existing encoded cluster. In order to avoid, as best as possible, clustering algorithms from incorrectly encoding triangles from two different roof planes into a single category, the vigilance parameter for all of the clustering algorithms has been set has high as needed to avoid this mishap. At the same time, the vigilance parameter, for each algorithm, has not been set exceedingly high as to cause the algorithm to mistakenly create extraneous categories when instead it was possible for the algorithm to realize encoding a given complex building's roof plane with fewer categories.

C. Preconditioning Techniques

Several preconditioning strategies were implemented to optimize the performance of the chosen clustering algorithms. In all cases considered, the data has been translated from its original location and relocated such that it was centered at the origin. The reasoning for doing this is explained in [9] in Section 3.2 on pages 4 and 5. The Cartesian coordinates of the normal vectors for the triangles existent in the triangulated irregular data are converted to polar coordinates. For GA and FSART, it was found optimal to normalize each dimension of the normal vectors of the data set to a single standard deviation. For FA however, the data set was compliment encoded before being presented and clustered.

After the buildings were manually extracted, only the roof triangles corresponding to those buildings were presented to the clustering algorithms. The building roof triangles were isolated utilizing the technique proposed in [9], [10].

IV. EXPERIMENTAL RESULTS

In order to compare the ability of the selected clustering algorithms, some performance measures are defined:

1. *Category Proliferation* – The number of categories it takes a clustering algorithm to encode a single roof plane should be minimal. Ideally, only one category will be created for each roof plane and that category will contain all triangles which are encompassed by that corresponding building roof plane.
2. *Encoding Accuracy* – Triangles belonging to a given cluster should only exist on a single roof plane (as opposed to a single cluster spanning across multiple roof planes).

A. Proposed Category Proliferation Measure (CPM)

When the vigilance parameter is set relatively high, extraneous categories are created for outlying triangles possessing characteristics significantly different from roof clusters established from a given clustering algorithm. A histogram was created of all of the occurrences all cluster labels have in a given clustered, triangulated LiDAR building roof. The cluster labels were then arranged corresponding to number of occurrences from greatest to least. The minimal number of the largest clusters which have a total number of occurrences that encode at least 50% of the total number of roof triangles (Tr) are considered significant clusters (Cr). If this number is less than the total number of roof planes (Pr), then the minimal number of significant clusters is equal to the total number of roof planes known apriori. For example, if the top two largest clusters have a total number of members consisting of 45% of the data set, but the top three consist of 60%, then the top three largest clusters are considered significant clusters. If a single cluster encodes 60% of the data set, but two clusters exist and two roof planes exist, then the top two largest clusters are considered significant clusters. All clusters not labeled as significant clusters are in turn noise clusters and all triangles holding membership to those clusters are noise triangles (Tn). After clustering, a planar regression analysis is done on the largest clusters existent in the clustered TIN. After a best fit plane is formed from that largest cluster, points from other clusters falling below a certain threshold are then merged to that cluster. It is therefore imperative that for each given roof plane, at least one cluster is existent in which the majority of its members (80%) are encompassed in a single roof plane. The following category proliferation measure (CPM) is therefore proposed to evaluate a clustering algorithm's encoding performance for a given LiDAR TIN:

$$CPM = \left[1 - \left(\frac{T_r - T_n}{T_r}\right) \cdot \left(\frac{P_r}{C_r}\right)\right] \cdot 100\% \qquad \textbf{(3)}$$

Ideally, all triangles will belong to significant clusters (Cr), therefore Tn = 0 and the number of roof planes (Pr) will equal the number of clusters (Cr) created by a given clustering algorithm. Therefore the clustering algorithm's performance under ideal conditions would cause the product of the ratios to equal 1 and then the subsequent CPM to equal 0. If a given algorithm performs poorly, a significant number of noise triangles may be created (Tn > 0) and therefore the ratio of (Tr-Tn)/Tr will decrease. Furthermore, the number of clusters created may be greater than the number of roof planes existent, therefore the ratio of Pr/Cr will decrease. The product of these ratios will therefore be relatively low and therefore the CPM will in turn be close to 1.

Noise will exist in the data set due to systematic errors; therefore small categories encoding these outliers will be created. This behavior is not penalized in the CPM as severely as the dependency of an algorithm to create numerous significant clusters to realize the encoding of a single roof plane. For the post processing planar regression analysis, it is optimal to have the majority of a given roof plane encompassed by a single cluster.

B. Proposed Encoding Correctness Measure

For a given clustered TIN, if a cluster (Cr), containing a substantial number of triangles, has 80% of its total triangles belonging to a single roof plane and no more than 20% of its total triangles spanning across multiple roof planes, the number of triangles of spanning across other roof planes is totaled and defined as (Ti). If the cluster has less than 80% of its total triangles confined to a single roof plane, then all of that cluster's triangles are totaled as Ti and the entire cluster is subsequently labeled as an incorrectly encoded cluster. The reason for this is, often times noise will cause a triangle belonging to one roof plane to have a similar orientation and perpendicular distance to the origin as a group of other triangles. Therefore, the noisy triangle is classified as incorrect instead of the entire cluster, provided that the majority of the cluster (80%) is still confined to a single plane. Eighty percent is used because a significant majority of the triangles must be correctly contained to the appropriate roof plane for planar regression and other post processing techniques to successfully extract planes and accurately construct 3D models from those planes. The Encoding Correctness Measure (ECM) is therefore proposed and defined as follows:

$$ECM = \left(\frac{T_r - T_n - T_i}{T_r - T_n} \right) \cdot 100\% \qquad \textbf{(4)}$$

Note that as the number of incorrect triangles (Ti) increases, the ECM approaches 0. If there are no incorrect triangles, then ECM = 100%. As the ECM decreases, the number of significant clusters correctly encoding the roof planes decreases, and therefore the success rate for 3D reconstruction from planar regression of those clusters also decreases. The ratio is defined as the number of correctly encoded triangles over the total number of non noise triangles.

C. Clustering Algorithm Parameter Settings

Each building was presented to each clustering algorithm (FA, FSART and GA) several times while the user defined parameters were adjusted. The parameters for a given clustering algorithm which yielded the best generalized results for that algorithm for all buildings were then noted and then those cases were presented. The purpose of this effort is to find a given unsupervised learning algorithm in which will encode all coplanar triangles with as few categories as possible and yet simultaneously retain the highest accuracy possible when clustering those triangles into groups corresponding to the actual depicted building roof planes. A single set of user defined parameters must achieve optimal results in a given algorithm for all structures as it would be impractical to have to change the clustering algorithm user defined parameters for each individual building. Below is a table of the user defined parameters found to yield the best results for all buildings tested:

TABLE 1
Clustering Algorithm User Defined Parameters

Algorithm Name	Choice Parameter	Learning Rate	Vigilance
FA	0.0001	0.1	0.8
		Gamma	Vigilance
GA		2	0.3
		Learning Time	Vigilance
FSART		100	0.2

Fuzzy SART's execution was terminated after 5 epochs. In all cases considered, if FSART was allowed to run for beyond 5 epochs, the CPM wound up rising with a slight decrease in the ECM. Conversely, if FA ran for at most 50 epochs with a low learning rate, FA learned the categories with more efficiency (higher ECM and low CPM). GA's performance was found to be better with Gamma initialized to two and a relatively low vigilance. GA's performance seemed optimal as long as the algorithm converged before 10 epochs (otherwise the algorithm was terminated at 10 epochs).

The characteristics of the three buildings tested are listed below:

TABLE 2
Building Parameters

Building #	Points	Roof Planes	Triangles
1	819	2	1620
2	3976	2	7941
3	1630	2	3242

Aerial digital photographs corresponding to those buildings are shown as follows:

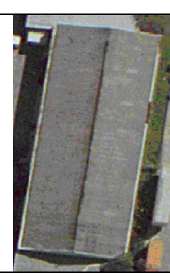
Fig. 4 – Building 1


Fig. 5 – Building 2


Fig. 6 – Building 3

The CPM and ECM evaluating FA, FSART and GA's performance on all of the above buildings in terms of grouping coplanar roof triangles is shown in the following three figures:

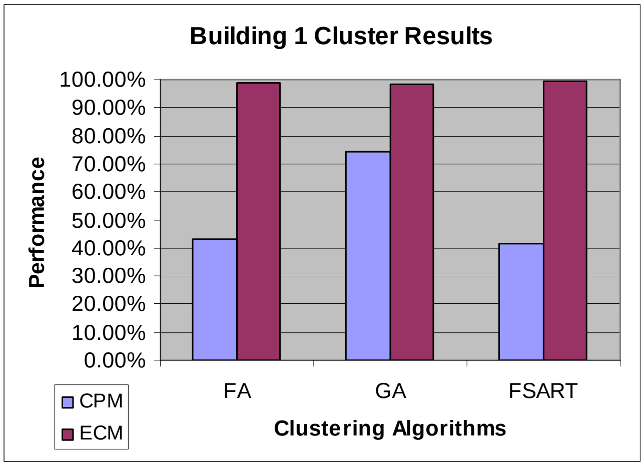

Fig. 7 – Building 1 Cluster Results

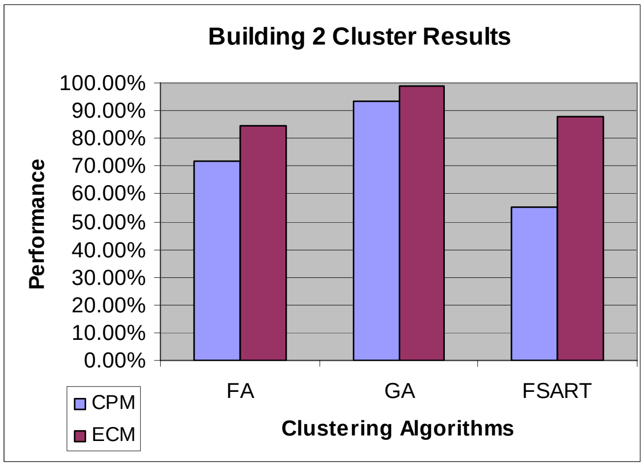

Fig. 8 – Building 2 Cluster Results

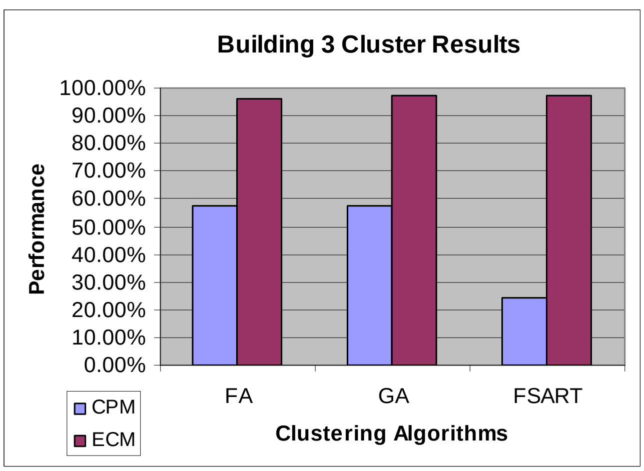

Fig. 9 – Building 3 Cluster Results

Using the results from the FSART clustering, the three buildings then underwent planar regression and were then reconstructed, all of which are shown in the following figures:

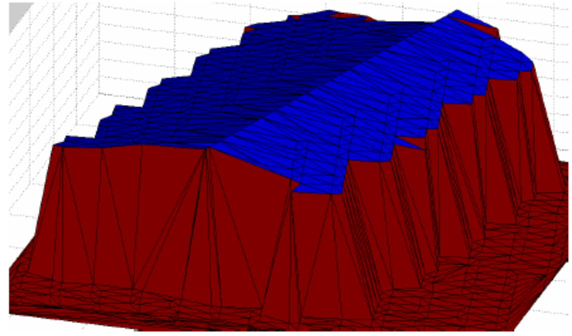

Fig. 10 – Building 1 Reconstructed

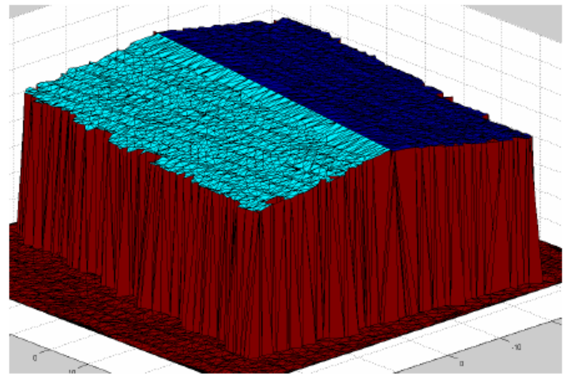

Fig. 11 – Building 2 Reconstructed

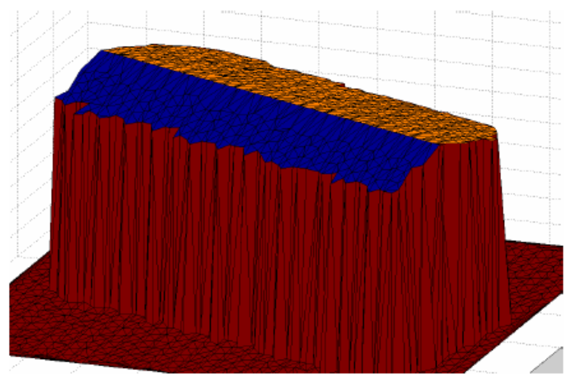

Fig. 12 – Building 3 Reconstructed

For additional reconstructed results see [9], [10].

V. Conclusion

For building #2, the high CPM's are due to the fact that the point density for that particular building was significantly higher than the other two buildings - due to overlapping flight paths. In all three cases, FSART achieved an equal if not lower CPM while retaining a competitive ECM in comparison to FA and GA. The reason for this is FSART encodes its categories with hyper spherical arcs. The normal vectors of a given roof plane, converted to spherical coordinates, all exist in a spherical arc diverging away from the origin. The width of that arc is in turn wider as the group of normal vectors becomes further saturated with noise. It takes both FA and GA more categories to accurately encode these normal vector roof clusters with hyper cubes and Gaussian distributions respectively. Hence FSART emerges the clear cut winner, in encoding the buildings with a CPM from 2% to as much as 35% less than the other algorithms. The lower the CPM, while still retaining a competitive ECM, allows the planar regression post process to form a best fit initial plane from a cluster with a higher number of correctly classified members, therefore resulting in a more accurately constructed plane.

One aspect in which this work can be continued further is rather than using existing algorithms which have certain strengths optimized for this clustering task, it would be even more efficient to derive an algorithm specifically tailored for this work – clustering coplanar triangles in a TIN derived from LiDAR. Another manner in which the above work can be furthered is to include additional features (as available) for clustering other than simply the data points themselves. Often, an aerial photograph accompanies a LiDAR data set. The colors, in the aerial photograph, corresponding to the triangle locations are features in which also could be included into the cluster analysis. Furthermore, current LiDAR sensors are also capable of capturing the returned intensity of the laser pulse emitted from the LiDAR sensor – another feature in which could be used to further distinguish the existent coplanar triangle clusters. There are several building detection algorithms existent in the literature. This work could also be continued by optimizing an existing or developing a new algorithm to automatically extract and present an increased number of isolated buildings for clustering.

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